Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Thursday **7 JUNE 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

Take $g = 9.8\ \mathrm{m\,s^{-2}}$ unless otherwise instructed.

The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 A racing car starts off down a straight section of track towards the first corner. Its speed, $v\ \mathrm{m\,s^{-1}}$, is modelled for the first four seconds of its motion by

$$v = t^3 - 9t^2 + 24t, \qquad 0 \leqslant t \leqslant 4.$$

(i) Find an expression for the distance travelled by the car in the first t seconds.

Calculate the distance travelled from $t = 2$ to $t = 4$. [5]

(ii) Show that the acceleration, $a\ \mathrm{m\,s^{-2}}$, of the car at time t is given by $a = k(t-2)(t-4)$, where k is a constant to be determined. [2]

(iii) Sketch the graph of a against t for $0 \leqslant t \leqslant 4$, stating the coordinates of the points where the graph crosses the a-axis and the t-axis. [3]

For $0 \leqslant t \leqslant 4$, calculate

(*A*) the greatest speed of the car, [2]

(*B*) the greatest acceleration and the greatest deceleration of the car. [3]

2

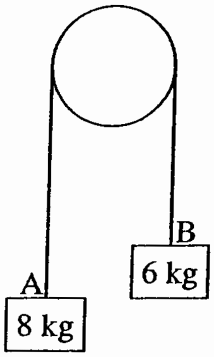


Fig. 2

A light inextensible string AB passes over a smooth peg. Particles of mass 8 kg and 6 kg are attached to the ends A and B of the string and hang vertically, as shown in Fig. 2.

The system is released from rest.

(i) Draw separate diagrams showing the forces acting on the particles at A and at B.

Write down the equation of motion for the particle at A and the equation of motion for the particle at B.

Show that the acceleration of the system is $1.4\ \mathrm{m\,s^{-2}}$.

Calculate the tension in the string. [7]

In the remaining parts of this question, assume that the particle at B does not reach the peg.

The particle at A falls 3 m before hitting the ground. This particle then stays on the ground.

(ii) How fast is the particle at A travelling when it hits the ground? [2]

(iii) How much further does the particle at B rise after the string goes slack? [2]

(iv) How much time elapses after the system is released before the string becomes taut again with the particle at B descending? [4]

[Turn over

3 In this question, the unit vector **i** is horizontal and the unit vector **k** is vertically upwards. All forces are in newtons.

A small, heavy box is suspended in mid-air and is held in equilibrium by the tension in a light inextensible string and a horizontal force, as shown in Fig. 3.

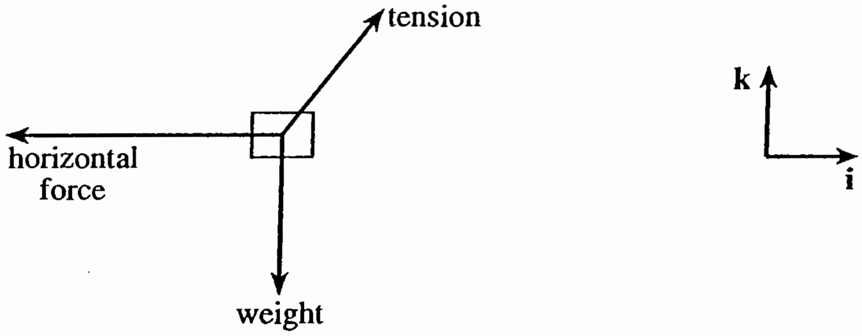


Fig. 3

The tension in the string is $\mathbf{T}_1$, where $\mathbf{T}_1 = 30\mathbf{i} + 49\mathbf{k}$. The horizontal force is $\mathbf{F}_1$, where $\mathbf{F}_1 = p\mathbf{i}$.

(i) Write down the value of p and show that the mass of the box is 5 kg. [3]

(ii) Calculate the magnitude of $\mathbf{T}_1$ and the angle $\mathbf{T}_1$ makes with the horizontal. [3]

Another force $\mathbf{F}_2 = 48\mathbf{i} - 87\mathbf{k}$ is now applied to the box. The force $\mathbf{F}_1$ still acts and the box is still in equilibrium.

(iii) The new tension in the string is $\mathbf{T}_2 = a\mathbf{i} + b\mathbf{k}$. Calculate the values of a and b. [3]

The tension in the string now becomes $\mathbf{T}_3 = q(13\mathbf{i} + 84\mathbf{k})$, where q is a positive constant. The magnitude of this tension is 340 N. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ remain unaltered.

(iv) Find the value of q. [3]

Find also the acceleration of the box in $\mathrm{m\,s^{-2}}$, giving your answer in the form $c\mathbf{i} + d\mathbf{k}$, where c and d are to be determined. [3]

4

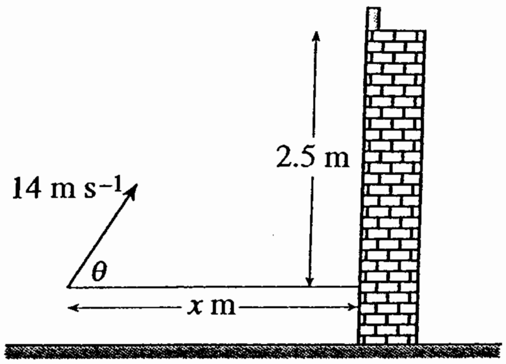


Fig. 4

A boy is firing small stones from a catapult at a can on a wall. All of the stones are projected with a speed of 14 $\mathrm{m\,s^{-1}}$ and at a vertical distance of 2.5 m below the can, as shown in Fig. 4. The boy varies his horizontal distance from the wall, x m, and the angle of projection of the stones with the horizontal, θ.

Standing in one position he finds that, when $\theta = 30°$, the stone hits the can $\frac{5}{7}$ seconds after projection.

(i) Calculate x. [2]

(ii) Show that the stone is at the top of its trajectory when it hits the can. [3]

He moves to the position where $x = 4$ and finds that there are two angles of projection which allow him to hit the can.

(iii) Write down expressions for the vertical and horizontal displacements of the stone at time t.

Use them to show that

$$4\tan^2\theta - 40\tan\theta + 29 = 0.$$

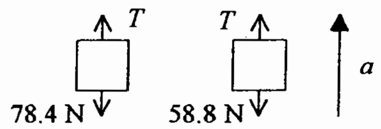
Hence find the two values of θ. [10]

Mark Scheme

1. (i)	$\int(t^3 - 9t^2 + 24t)dt$	M1 Use of integration (at least 2 indices correct)
	$= \frac{1}{4}t^4 - 3t^3 + 12t^2 + C$	A1 At least two terms correct (neglect constant of integration)
	$s = 0$ when $t = 0$ so	
	$s = \frac{1}{4}t^4 - 3t^3 + 12t^2$	A1 All terms correct + evidence of arb const considered or correct limits for definite integral
	Dist travelled from $t = 2$ to $t = 4$	
	is $(64 - 192 + 192) - (4 - 24 + 48)$	M1 Evaluation of arb constant (or subst of limits) in some expression obtained by integration.
	$= 36$ m	A1 cao [5]
(ii)	$a = 3t^2 - 18t + 24$	M1 Use of differentiation, at least 2 indices correct
	$= 3(t^2 - 6t + 8) = 3(t-2)(t-4)$	E1 All correct. Accept $k = 3$ implied or seen [2]
(iii)	Sketch showing parabola thro' (0, 24), (2, 0) and (4, 0)	B1 shape B1 (0, 24) dep 1st B1. FT **their** k. Accept $(0, 8k)$ B1 (2, 0) and (4, 0) dep 1st B1 [Accept the 24, 2 and 4 clearly marked on the axes] [SC: Polygon drawn B0 B1 B1] [3]
	(A)	
	Max speed at SP or boundaries	
	$a = 0$ gives $t = 2$ or $t = 4$	B1 Must have both roots or max speed implied by max area i.e. when $t = 2$ or max demonstrated
	$(v(0) = 0)$, $v(2) = 20$, $v(4) = 16$	
	Hence greatest 20 m s^{-1}.	A1 [Award this mark if seen WW] [2]
	(B)	
	Local min of a is when $t = 3$	B1 (Allow WW) Award if correct decel seen
	greatest acc is 24 m s^{-2} at $t = 0$	B1 FT **their** (0,24) if awarded above. Do not accept $8k$
	greatest decel is 3 m s^{-2} at $t = 3$	B1 no FT accept $a = -3$ [3]

[Total 15]

2. (i)



B1 Diagram. Accept any form for weight. Arrows required. Acc not required. Accept different tensions only if shown equal later. Accept equivalent single diagram. No spurious forces.

For A, using N2L $8 \times 9.8 - T = 8a$	M1 Use of N2L. Allow '$F = mga$' and sign errors. condone one force missing. Accept spurious forces A1 LHS correct. Accept only sign error on RHS.
For B, using N2L $T - 6 \times 9.8 = 6a$	A1 Must be consistent with equation for A. Signs consistent, all forces present and *ma* used
Either	
Solving	M1 Elimination of T or a.
$a = 1.4$ so 1.4 m s^{-2}	E1
Tension 67.2 N	F1 FT wrong equations only. [If given value used for a and **both** equations checked, M1 E1 F1, otherwise M0 E0 F1] [7]
Or	
Applying N2L 'overall' to find T and a. $(8-6) \times 9.8 = (8+6)a$	M1 Use of N2L. Allow '$F = 14ga$' as only error
$a = 1.4$ so 1.4 m s^{-2}	E1
Tension is 67.2 N	F1
(ii) $v_A^2 = 2 \times 1.4 \times 3 = 8.4$ $v_A = 2.89827..$	M1 Use of '*uvast*' with **their** accn (**not** g) A1 cao. [SC 1 for correct answer from wrong or unclear working] [2]
(iii) $0 = 8.4 - 2 \times 9.8 \times s$ $s = 0.4285..$ so 0.429 m (3 sf)	M1 Use of appropriate '*uvast*' with g. A1 FT from **their** v_A. [SC 1 for correct answer from wrong or unclear working] [2]
(iv)	M1 Using appropriate acceleration in each phase Accept **their** $a = 1.4$
First part $2.8982.. = 0 + 1.4t$	B1 FT their v_A etc but **not** their a
Second part $2.8982.. = -2.8982.. + 9.8t'$	B1 FT their v_A etc or equivalent if other method
Total is 2.661.. so 2.66 s (3sf)	A1 cao [4]

[Total 15]

3. (i)	$p = -30$	B1 Accept $p = -30\,\mathbf{i}$	
	The weight of the box is 49 N The mass is $\frac{49}{9.8} = 5$ kg	M1 Use of the **k** component E1 Accept $\frac{49}{g}$.	[3]
(ii)	Magnitude is $\sqrt{30^2 + 49^2}$ = 57.454... So 57.5N (3 s. F.)	B1	
	Direction is $\arctan(\frac{49}{30})$	M1 Give for $\arctan(\frac{49}{30})$ or $\arctan(\frac{30}{49})$ seen	
	= 58.523.. So 58.5° (3 s. f.)	A1 Accept 121.5 °.	[3]
(iii)	$\mathbf{T}_2 + (48\,\mathbf{i} - 87\,\mathbf{k}) - 30\,\mathbf{i} + (-49\,\mathbf{k}) = \mathbf{0}$	M1 Correct equilibrium statement with signs correct in one component. (If 2 eqns used, signs correct in 1 of them). Accept omission of -49 **k**.	
	$\mathbf{T}_2 = (-18\,\mathbf{i} + 136\,\mathbf{k})$ $a = -18$ $b = 136$	A1 FT only from wrong $\mathbf{F}_1$. Accept -18 **i** A1 cao. Accept 136 **k**	[3]
(iv)	Magnitude of $\mathbf{T}_3$ is $q\sqrt{13^2 + 84^2}$	M1 Any attempt to find the magnitude of $\mathbf{T}_3$ or of 13 **i** + 84 **j**.	
	so $340 = 85q$ $q = 4$	A1 for the 85 A1 Award if $q = 4$ seen and A1 above not awarded	[3]
	$52\,\mathbf{i} + 336\,\mathbf{k}$ $- 30\,\mathbf{i}$ $+ 48\,\mathbf{i} - 87\,\mathbf{k}$ $- 49\,\mathbf{k} = 5\,\boldsymbol{a}$	M1 Use of N2L , must have vector form and 5**a** or correct magnitude of acceleration	
	so $\mathbf{a} = (14\,\mathbf{i} + 40\,\mathbf{k})$ m s^{-2}	A1 cao A1 cao each term	[3]

[Total 15]

4. (i) $x = 14\cos 30 \times \frac{5}{7}$	M1 Use of horiz cpt of distance with $a = 0$ Accept sin $\rightleftharpoons$ cos
$= 5\sqrt{3}$ or 8.66 (3 s.f.)	A1 [2]
(ii) Using $\dot{y} = 14\sin 30 - 9.8 \times \frac{5}{7}$	M1 Use of vert cpt of speed or distance, $g = \pm 10, \pm 9.8(1)$. Accept sin $\rightleftharpoons$ cos
$= 0$	A1 Correct subst including time, cpts and signs E1 Clear conclusion [3] [If numerical approach; Subst 2 values in range $0.713 \le t \le 0.715$ or better, one each side of $\frac{5}{7}$ or use symmetry of the parabola M1 A1 Compare with $y = 2.5$ and clear conclusion E1]
(iii) $y = 14\sin\theta t - 4.9t^2$	B1 Or 2.5 = ...
$x = 14\cos\theta t$	B1 Or 4 = ...
$t = \frac{x}{14\cos\theta}$, $y = 2.5$, $x = 4$	M1 Eliminating t B1 Substituting values for x and y (this may be done in the equations above)
so $2.5 = 14\sin\theta \times \frac{4}{14\cos\theta} - 4.9(\frac{2}{7})^2\sec^2\theta$	B1 Correct substitution . Accept $\frac{1}{\cos^2\theta}$.
$\Rightarrow 2.5 = 4\tan\theta - 0.4(1 + \tan^2\theta)$	M1 Use of $\sec^2\theta = 1 + \tan^2\theta$
$\Rightarrow 4\tan^2\theta - 40\tan\theta + 29 = 0$	E1 NB as given
$\tan\theta = \frac{40 \pm \sqrt{40^2 - 4\times 4\times 29}}{8}$	M1 Solving for $\tan\theta$ (accept GDC).
$\tan\theta = 0.7869.., 9.213....$	A1 Either correct
Angles are 38.2°, 83.8° (3 s.f.)	A1 Both correct [10] [for last 3 marks award G3 both correct to at least 3 s.f. WW or G2 for one correct to at least 3 s.f. WW]

[Total:15]

Examiner’s Report

Mechanics 1 (2607/1)

General

The paper seemed to be of about the correct length and there were many good answers seen to every question. However, many of the candidates struggled to deal with any of the questions, leading to the impression that the paper was at about the right level of difficulty with an appropriate amount of guidance for only a minority of them. The candidates seemed either to do well or very poorly. Many candidates in the latter category scored very few marks; apart from difficulties with even the easiest parts of the questions, they seemed to have poor general mathematical skills so that they were unable to recognise such basics as the form of a quadratic graph or could not solve a quadratic equation using the formula. There was also an impression that some had inadequate preparation, at least in terms of lack of practice, and many were not familiar with the extra syllabus items on projectile motion not found in the 5507 specification. All of this would be entirely consistent with the lack of time to prepare for this examination reported by many centres.

The better candidates produced some excellent scripts and many of these presented their answers well by, for instance, making it clear in which direction positive was taken when using standard results from kinematics.

Question 1 (Kinematics applied to a racing car)

There were a large number of candidates who scored zero on this question as they considered acceleration to be constant and hence applied the uniform acceleration formulae throughout. Putting these candidates to one side, common errors were as follows.

In part (i), many candidates used differentiation instead of integration. Most of the integration done was correct but many candidates failed to consider the constant of integration – either neglecting the term or failing to evaluate it given the initial conditions. Those candidates who attempted to find the distance travelled between $t = 2$ and $t = 4$ usually obtained the correct answer although some merely evaluated the separate displacements at these times and did not subtract to find the distance.

Part (ii) was usually done reasonably well except by the candidates who were weak overall. A common error was to differentiate correctly and then to divide by 3 to obtain an expression equivalent to $(t-2)(t-4)$; k was then taken to be 1 or not stated.

In part (iii), a significant number of candidates drew 'quadratic polygons', others thought the form of the graph implied a cubic curve (or higher order polynomial or even a sine curve). Many, however, were able to score 3 marks quite easily. In (A), it was quite common to see $t = 2$ considered alone without the other time at which $a = 0$ $(t = 4)$; there was a penalty for this unless a good reason was given. In (B), many candidates realised that the greatest deceleration occurred when $t = 3$ but some forgot to state its value.

[(i) $s = \frac{1}{4}t^4 - 3t^3 + 12t^2$, 36 m; (ii) $k = 3$; (iii) (A) 20 m s^{-1}; (B) 24 m s^{-2}, 3 m s^{-2}]

Question 2 (Motion of Connected Particles)

In part (i), diagrams were often good although some candidates indicated the weights on each particle as 'mg' or 'W'. A particularly common error was to label the tensions as, say, T_1 and T_2 and then not to realise that they were equal; such candidates often went on to give two (different) tensions in the string. Equations of motion were attempted by most of the candidates although it seemed that many had not encountered this routine question before and some did not seem to understand the term; some omitted tensions, others wrote only one side of each equation and others did not seem to be applying Newton's second law at all. However, there were many completely correct solutions to this part enabling candidates to score 7 marks very readily. A large number of candidates established the given common acceleration of 1.4 m s^{-2} by considering the system as a whole 'around the pulley'. This method was accepted but such candidates were often unable correctly to find the tension in the string.

Part (ii) was well answered by the majority of the candidates.

Part (iii) caused difficulties as many candidates continued to use the common acceleration of the particles instead of the acceleration due to gravity when B becomes a projectile.

Correct answers to (iv) were seldom seen. This was usually due to poor organisation. Candidates failed to use the appropriate acceleration or calculated the time taken for only one phase of the motion or forgot that B moves down as well as up before the string again becomes taut.

[(i) 67.2 N; (ii) 2.90 m s^{-1} (3 s. f.); (iii) 0.43 m (3 s. f.); (iv) 2.66 s (3 s. f.)]

Question 3 (Vectors expressions for forces in 2 dimensions acting at a point)

In part (i), it was more common to see the incorrect response of $p = 30$ than the correct one of $p = -30$. Although many of the candidates were able to show that the mass of the box is 5 kg, others tried to involve both components and some of the reasoning was poorly expressed; statements of the form '$Wg = 49$ so $W = 5$ kg' were common.

Part (ii) was done very well by the majority of the candidates.

Problems occurred in part (iii), similar to those in part (i), because many candidates attempted to treat the forces in *vector form* as if they had been given a diagram with magnitudes and directions; it was common to see statements such as $\mathbf{F_1} = \mathbf{T_2} + \mathbf{F_2}$ and few candidates wrote a vector equation of the form $\sum \mathbf{F_i} = \mathbf{0}$. Some successful candidates wrote down two separate equilibrium statements.

In part (iv), it was pleasing to note that many otherwise weaker candidates were able to find the correct value of q. There were, however, many basic errors such as $340 = q(13\,\mathbf{i} + 84\,\mathbf{k}) = 97q$ or $340 = \sqrt{13q^2 + 84q^2}$. The final part of this question was completed successfully by only a few candidates. There were errors similar to those in part (iii) in finding the resultant force, some candidates only applied Newton's second law in one direction (usually incorrectly) and others just attempted to find the resultant force and set this equal to **a**. Some credit was given to the candidates who successfully found the correct magnitude of the acceleration.

[(i) -30; (ii) 57.5 N (3 s. f.), 58.5° (3 s. f.); (iii) $a = -18$, $b = 136$; (iv) $q = 4$, $\mathbf{a} = (14\,\mathbf{i} + 40\,\mathbf{k})$ m s^{-2}]

Question 4 (Projectiles)

Part (i) was done quite well by many candidates but others seemed to have no idea how to solve such problems; there were many candidates who thought that they could consider the motion as if were in a straight line at the angle of projection which could then be resolved into horizontal and vertical components.

In part (ii), many candidates realised that they needed to show that the vertical component of velocity was zero at the given time but some failed to make their argument explicit. A significant number of candidates merely showed that when $t = \frac{5}{7}$, the upward displacement is 2.5 m which, of course, was given in the question and could occur at *any* point on the trajectory.

In part (iii), candidates often did not give their expressions for the horizontal and vertical displacements at time t in the form $x = \ldots$ and $y = \ldots\ldots$, or equivalent forms. It was quite common to see $s = 14t\cos\theta$ and $s = 14t\sin\theta - 4.9t^2$ which are meaningless in terms of parametric equations and the associated cartesian equation and sometimes seemed to halt any further progress from the candidate. Despite this, a significant number of candidates knew how to proceed and correctly eliminated t. However, many candidates then came to a halt because they did not know or recognise the need for the identity $\sec^2\theta = 1 + \tan^2\theta$. The two correct values of θ were found by the majority of the generally more successful candidates (whether or not they had established the given quadratic equation) but many candidates clearly did not know how to solve such an equation.

[(i) 8.66 (3 s. f.); (iii) 38.2°, 83.8° (3 s. f.)]